Activity description

In this activity students model data concerning the time caffeine from tea, coffee or cola stays in the body's blood stream.

This involves drawing graphs and finding functions and parameters to fit different models to the graphs.

Linear and exponential functions are the most appropriate.

Suitability

Level 3 (Advanced)

Time

2-4 hours

Resources

Student information and worksheets, spreadsheet

Optional: slideshow

Equipment

Graph paper, graphic calculators or computers.

Key mathematical language

Model, linear function, exponential function, parameter

Notes on the activity

This activity is written in a way that encourages students to work independently and so could be used as an assignment.

However some students will find it difficult to make decisions about which functions to use and how to approach the tasks. You could discuss this with them individually when necessary.

Alternatively you could use the graph on the slideshow to help in class discussion about which types of functions are suitable before students try to find particular functions for themselves.

The data is provided on a spreadsheet, but students could use a graphic calculator and/or do some work by hand if preferred.

It is expected that students will use an exponential function of the form $y = Ae^{-kt}$ to model the full data set and a linear function of the form, $y = mt + c$, for just the first part of the data set.

There are a number of ways in which students can find particular functions.

For example, they could

- find the intercept and gradient from their graph of the dataset
- use a graphic calculator or spreadsheet to plot a graph of the data, and then add a line/curve of best fit (or use trial and improvement) to find values for the parameters that give a good fit
- substitute the coordinates of data points into each function, then use algebraic techniques to find the corresponding values for the parameters.

For each data set there are a variety of exponential and linear functions which could be used.

During the activity

You could use one of the data sets to show students how to find an exponential model, and then divide students into groups to find exponential functions for the other data sets. The results could then be used in class discussion to consider how the values of A and k are related to the graph of $y = Ae^{-kt}$.

Points for discussion

You will need to decide to what extent you want to give students the opportunity to work independently.

You could give very little help, or use class discussion to identify what you expect students to do. For example, at the beginning of the activity you could discuss which types of functions could be used to model the data, and ask students to suggest how they can find the values of the parameters that would give a good fit.

At the end of the activity the results could be compared with other examples of exponential decay, for instance radioactive decay or the fall in temperature of a cooling object (perhaps a cup of coffee). This provides an opportunity for students to consider how the same mathematics is applicable over a range of situations.

Students could also compare the different methods (graphical and algebraic) for finding values for the parameters of the models.

Extensions

More able students can be encouraged to substitute the coordinates of data points into each function, then use algebraic techniques to find the corresponding values for the parameters.